Numerical study of buckling and flutter of clamped-clamped pipe carrying fluid under different parameters

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Keywords: Pipe carrying fluid, natural frequency, critical velocity, instabilities, range. elastic foundation, finite element method, and Matlab,

Abstract. This article aims to establish a coupled fluid-structure numerical model, for calculating the natural frequencies and critical velocities of a clamped-clamped pipe carrying incompressible fluid. The fluid circulating has the flexional motion as that of pipe structure. The equations are discritized with standard finite element method. We developed a program under Matlab. The advantage of Matlab language by using standard functions is to present the first eigen-modes of the system aspect interaction fluid-structure for different parameters in complex planes. The numerical approach is based on some research and analytical models. Numerical results show the effect of mass ratio, length and elastic foundation on instabilities regions and static instability range.

Introduction

The problem of pipes conveying fluid has very important role in various industrial applications. They are used in engineering industries, heating exchangers as nuclear production, pipeline, and hydropower systems. Predicting the various values prevents engineers and technologists from approaching or falling into the resonance phenomenon. The first study of the pipe dynamic under internal flow is found in Housner [1]. Then it is followed by researcher Païdoussis, who published a book [2] in which he collected his various researches and results, linear and non-linear motion equations using analytical method called Galerkin and experimental method as well as factors affecting this behavior as fluid velocity, mass ratio, elastic foundation, and gravity.

The book has become a reference for every researcher in this science. In the reference, the natural frequencies and the critical velocities of instability are determined in various boundary conditions for fluid-conveying pipe. Then studies continued on this pattern, we find the researcher Maalawi and al. [3]. Their studies had presented a mathematical model for determining the critical flow velocity of a pinned-pinned pipe composed of uniform modules, design parameters include the wall thickness and the length of each module. In a different sense, Doaré studied the role of boundary conditions in the instability of one-dimensional systems by utilizing a local wave equation [4].

Chellapilla and al [5], studied the effect of a Pasternak foundation on the critical velocity of a fluid-conveying pipe by Galerkin method. He repeated the same search using fundamental frequencies calculations of a pipeline resting on a two-parameter foundation with different boundary conditions [6]. Some studies have dealt with the thermal effect on instability such as that found by Qian and al [7], which studied the static instability of pinned-pinned fluid-conveying pipe under thermal loads. The equation of motion is derived for the straight pipe under the effects of linear and non-linear stress–temperature cases. In addition to the analytical methods, the numerical methods presented significant results in studying this behavior similar to the method of spectral element modeling [8].

Finite element analysis was used by Salah to analyze dynamically the stability of a pipe which is stiffened by linear spring and conveying an internal flow of fluid [9]. The same method was adopted in the references [10-13]. Dahmane and others used numerical simulation using the finite element method to calculate natural frequencies in terms of fluid velocity and for several factors [14]. In the present study, calculation methods have been developed for the analysis of instabilities regions and instability range in clamped-clamped pipe. Numerical modeling of structure-fluid was conducted by the finite element method (FEM).

The characteristics of static and dynamic instability, namely the disappearance of the first vibratory mode and the attainment of the maximum (critical) velocities associated with it, are carried out using a program developed on MATLAB. After studying the numerical approach, several examples were studied. We performed several calculations to obtain the critical velocities under various factors, taking into account: fluid velocity, mass ratio, length and elastic foundation. The results were presented by displaying the natural frequencies in terms of the flow velocity, which enables us to analyze the instabilities under these effects.

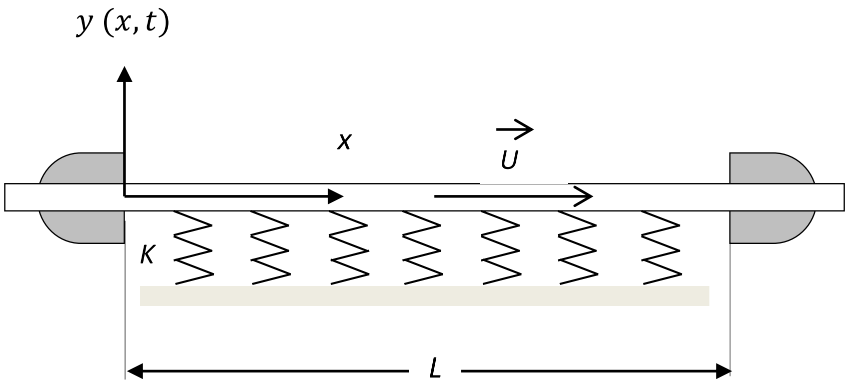


Fig. 1 Representation of the pipe carrying fluid resting on an elastic foundation Winkler-type.

The physical model of clamped-clamped pipe conveying fluid with Winkler-type is shown in **Fig. 1.** We consider a straight pipe with a length L, an internal cross-section area A, a mass per unit length m_s, and a flexural rigidity EI. The mass per unit length of a conveying fluid m_f has an axial velocity U varying with time, referring to a Cartesian coordinate system ($Oxyz$). The pipe is assumed as an Euler-Bernoulli beam initially aligned with the x axis and lateral displacement, where the motions are small ($dx \approx \delta s$). **Fig. 2 (a)** shows forces on fluid element while, **Fig. 2 (b)** shows forces and moment of pipe element. Linear pipe-dynamic equations on an elastic foundation have been derived in the previous work [2],

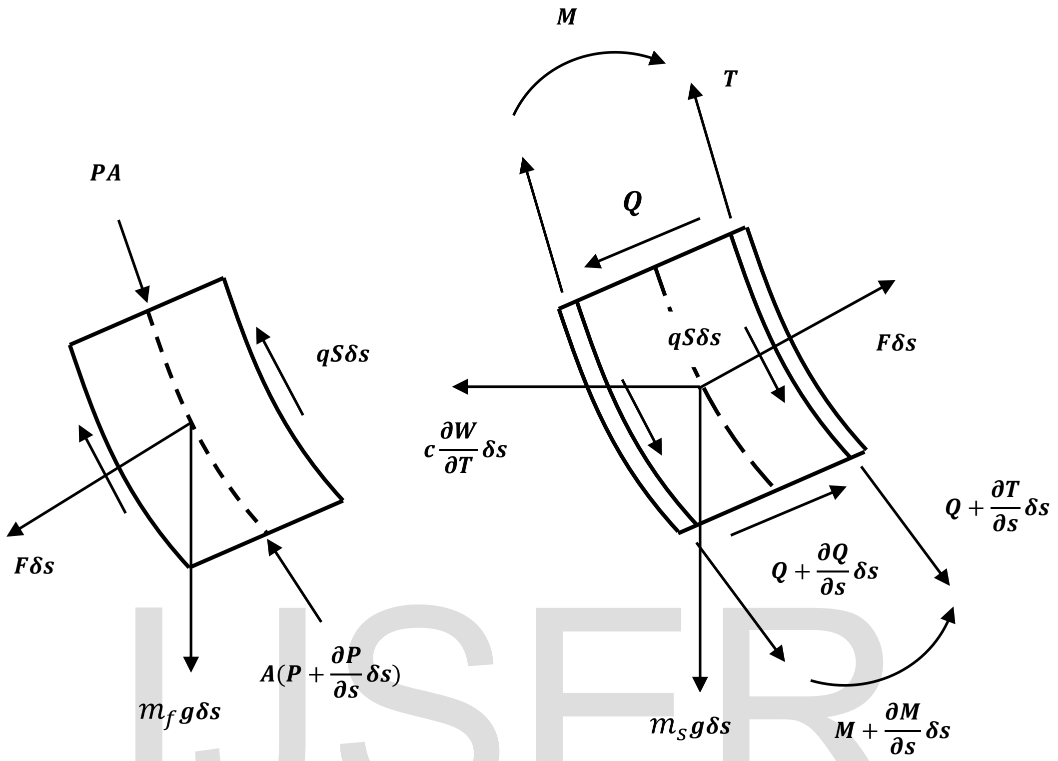


Fig. 2 (a) forces on fluid element; (b) forces and moments on pipe element δs [2].

$$EI\frac{\partial^4 y}{\partial x^4}+m_f U^2\frac{\partial^2 y}{\partial x^2}+2m_f U\frac{\partial^2 y}{\partial x\partial t}+\left(m_s+m_f\right)\frac{\partial^2 y}{\partial t^2}+Ky=0 \tag{1}$$

The boundary conditions for clamped-clamped pipe are,

$$y|_{x=0}=\frac{\partial y}{\partial x}|_{x=0}=y|_{x=1}=\frac{\partial y}{\partial x}|_{x=1}=0 \tag{2}$$

We use the same non-dimensional variables and parameters as in reference [2],

$$T=\left(EI/\left(m_f+m_s\right)^{1/2}\right)t/L^2,\quad \beta=m_f/\left(m_f+m_s\right),$$

$$u=UL\left(m_f/EI\right)^{1/2},\ k=KL^2/EI$$

Finite element discretization

The equation of element deflection for straight two dimensional beam elements could have the form [15], and [16],

$$W(x,t)=\sum_{i=1}^{N}N_i(x)W_i(t) \tag{3}$$

$[N_i]$: Represent the shape function,

$W_i(T)$: Is the function which represents the displacements shape and rotations.

Elementary matrices

The potential energy of the solid element can be expressed,

$$V_s = \frac{1}{2}\int_0^L EI\left(\frac{d^2 W}{dx^2}\right)^2 dx \tag{4}$$

The kinetic energy of the solid element can be expressed,

$$T_s = \frac{1}{2}\int_0^L m_s \frac{d^2 W}{dt^2} dx \tag{5}$$

The kinetic energy of the fluid element [11] can be expressed,

$$T_f = \frac{1}{2}\int m_f \left(U\frac{dW}{dx} + \frac{dW}{dt}\right)^2 dx \tag{6}$$

The potential energy over the length of Winkler elastic foundation [13] can be expressed,

$$V' = \frac{1}{2}\int_0^L KW^2 dx \tag{7}$$

The different elementary matrices are given in appendix A.

After using the Lagrange principle, the equation of motion by finite element method is,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + ([K])\{q\} = 0 \tag{8}$$

Where $[K]$ is the rigidity (global stiffness) of the system,

$$[K] = [K_s] - [K_f] \tag{9}$$

The solution of equation (8) is very complicated with presence of damping, so we use the variable change method (state-space),

$$E\dot{z} + Gz = 0 \tag{10}$$

Where the state variable is,

$$z = \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} \tag{11}$$

The resolution method has been presented in appendix B.

Results and discussion

In the current work, results will be discussed for various values of fluid velocity, mass ratio, length, and Winkler elastic foundation, calculating the frequency of the first three eigen-modes for fluid-conveying pipe, with finding the critical velocities of instabilities. The physical parameters as: elastic modulus of structure is 2.1*10^2 GPa; incompressible fluid, density is 10^3 kg/m^3; elastic structure, density is 7850 kg/m^3. The geometrical parameters are: pipe length is L belongs to [1, 2] m; the thickness corresponds β belongs to [0.1, 0.7]; outer diameter of the pipe is 3*10^{-2} m. The research in our hands has adopted a beam with the fixed-fixed boundary conditions because it is the most present in the industry, such as boilers, generators, and heat exchangers at high fluid velocity. This type of boundary conditions gives the system more rigidity and stability, so we use very high velocity to reach the instabilities. The cases can be divided into two; according to the parameters effect masse ratio β, length L and elastic foundation k-Winkler model.

Effect of masse ratio β

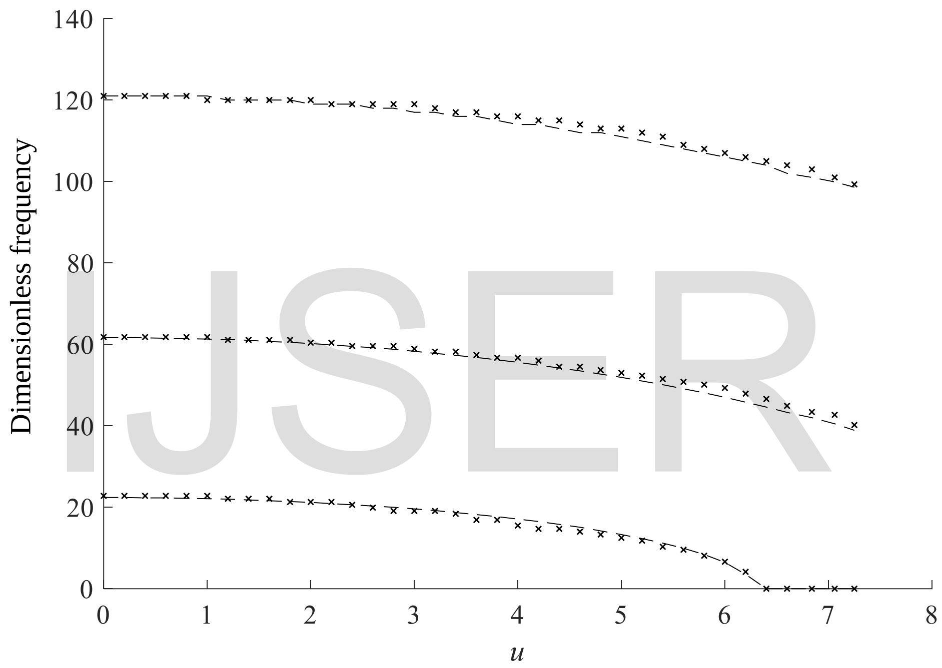


Fig. 3 Dimensionless frequency for various values of u, for the lowest three modes of a pipe carrying fluid, DTM [11] (xxx) and FEM (− − −), β = 0.1.

In the case one, and firstly, the numerical results are obtained by the differential transformation method (DTM) in reference [17], and FEM for masse ratio β = 0.1. The semi-analytical and numerical results are similar (figure 3). We repeated the numerical calculations for two different mass ratios, β = 0.3 in figure 4, and β = 0.5 in figure 5. In the same figures, part (a) presents the dimensionless (non-dimensional) results, and part (b) presents the dimensional (physic) results.

The dimensional results show that there is a great variation in the natural frequency developments, unlike what is found in the dimensionless frequencies, and this is contrary to what many researchers like Chellapilla and al. shown in reference [6]. For $U \equiv 0$ (no flow), the first frequency variation is 3 %. For $U \equiv U_{cr}$,where is the first critical velocity which corresponds to static instability (u = 6.384), we find the variation between the two cases is equal to 45%, while the dynamic critical velocity is reduced with 43.6% corresponding to flutter.

The variation of instability static range is 39.8%. So, an increase in the value of β leads to a decrease in the stability region, critical velocities, as well as the instability range. The effect of all this analysis is that damping in this type of behavior is positive, which leads with the passage of time to gradually reduce the rigidity of the system down to flutter.

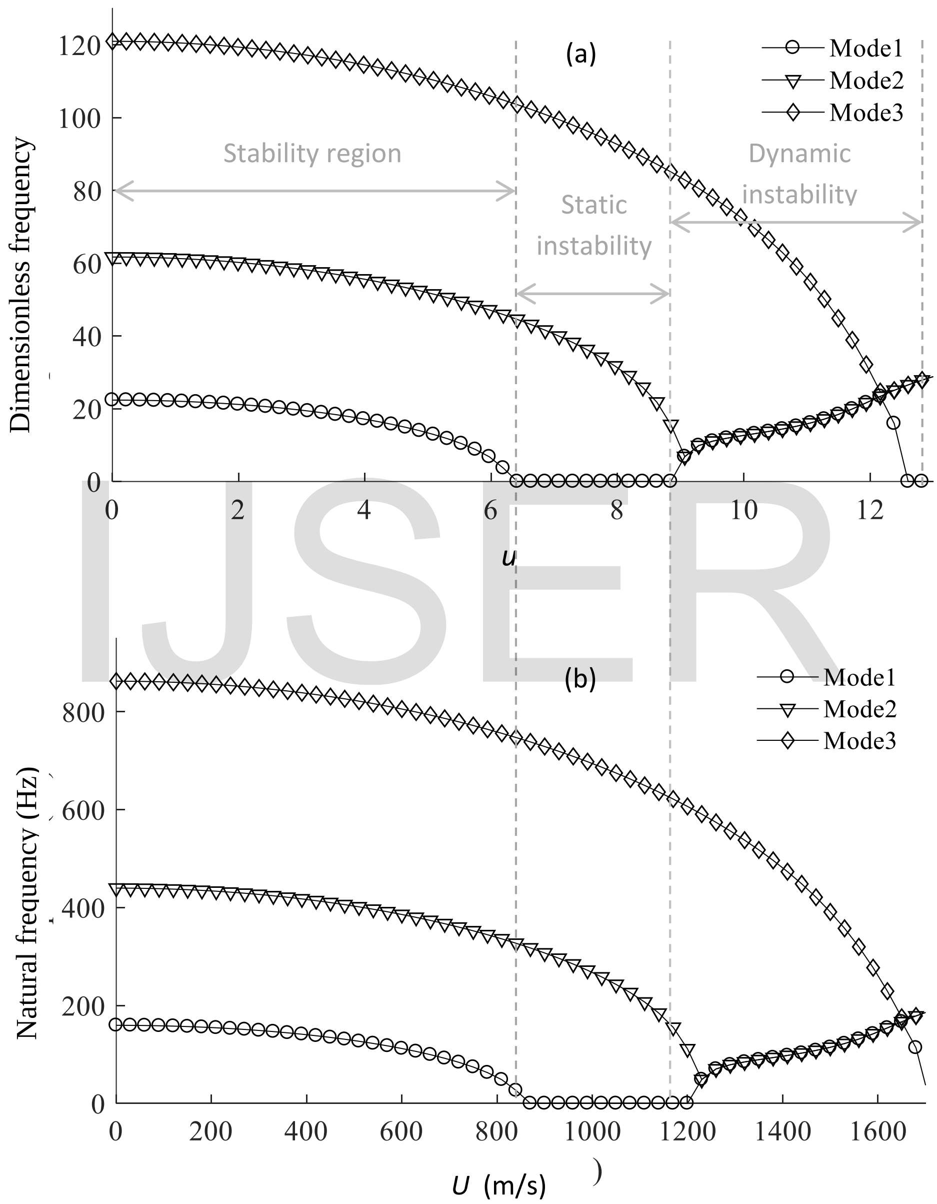
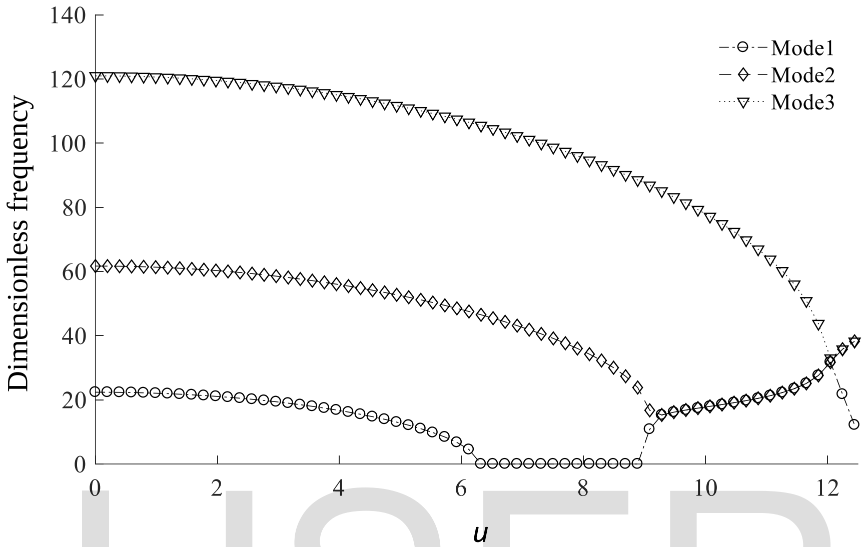
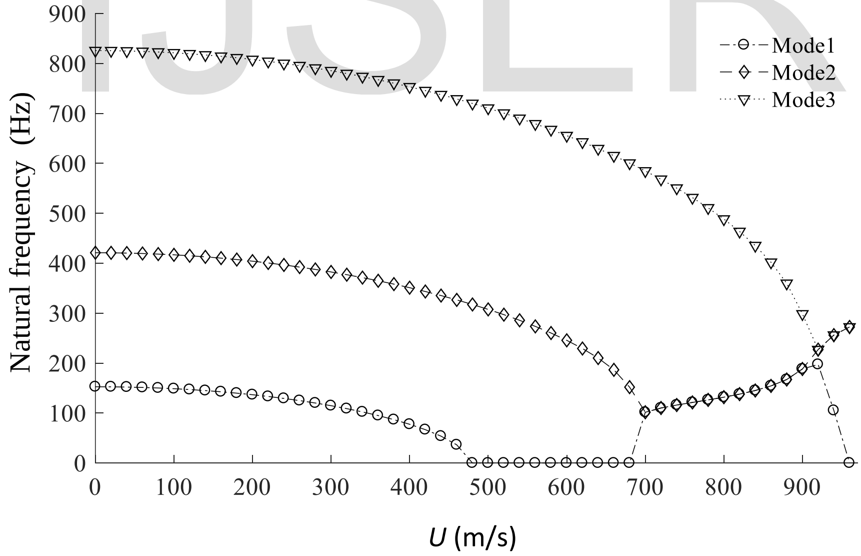


Fig. 4 Three proper modes on fluid velocity function of clamped-clamped pipe carrying fluid, β = 0.3, (a) dimensionless frequencies, (b) naturel frequencies (Hz).



(a)



(b)

Fig. 5 Three proper modes on fluid velocity function of clamped-clamped pipe carrying fluid, $\beta = 0.5$, (a) dimensionless frequencies, (b) naturel frequencies (Hz).

Effect of elastic foundation and length

In the next case, we carry out the same study with the addition of the effect of elastic foundation and length. For elastic foundation Winkler-model we will take two different values, a minimum value $k = 10$ and a maximum values $k = 10^3$. **Fig. 6**, and **Fig. 7** present the first three natural frequencies as a function of the fluid velocity of clamped-clamped pipe on a weak elastic foundation Winkler-type where $k = 10$, for three different lengths ($L = 1$, $L = 1.5$, $L = 2$), with $\beta = 0.3$, and $\beta = 0.5$, respectively. For $\beta = 0.3$ and $L = 1$ **(Fig. 6 (a))**, the biggest change does not exceed 1% for natural frequencies. While it exceeds 4% for the instability range, the first critical velocity is 6.389.

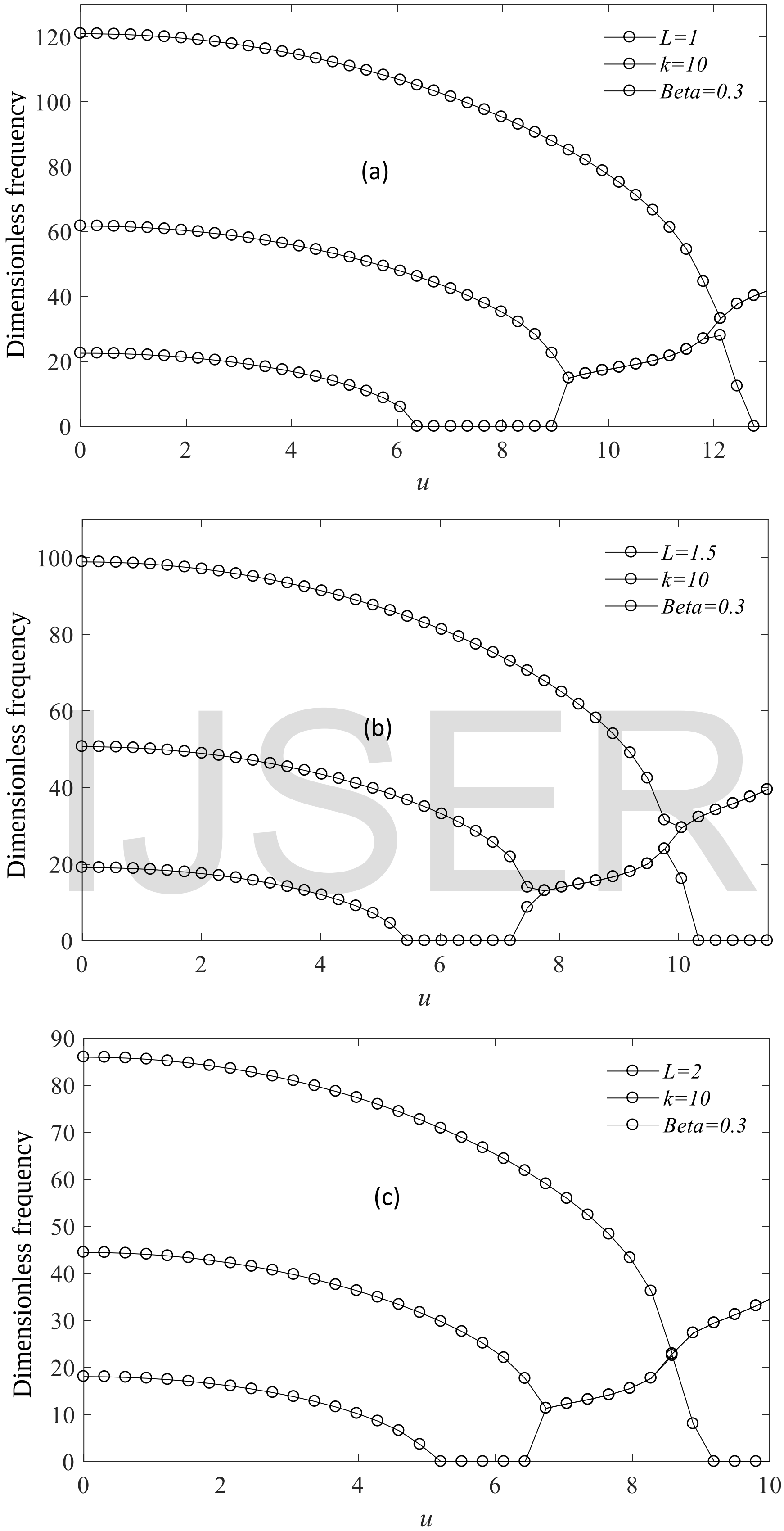


Fig. 6 Effect of length on the natural frequency of the clamped-clamped pipe on elastic foundation (k = 10) at different fluid velocities, β = 0.3.

The **Fig. 6 (b)** presents a large variation in the natural frequencies, so it is 16%. The critical velocity corresponding to the buckling decreases to a limit of 5.41, i.e. 15.33%, while the instability range value decreases to 36.8%. When increasing the value of the length to the double (L = 2), we find that the percentage of change in instability margin (range) diminishes to 40.9% compared to the first case (L = 1). When we raise the value masse ratio down to 0.5 as shown in the **Fig. 7.**

The instability range increases by 11% with respect to the same length, see **Fig. 7 (a)**, while the variation in the level of the first critical velocity and natural frequencies is almost non-existent. The **Figures 7 (b)** and **(c)** show the variations accompanying the increase in length. The **Figures 8 (a), (b)** and **(c)** show the variation in the natural frequencies and critical velocities when increasing the value of the elastic foundation to 10^3 and with different lengths for $\beta = 0.3$. The **Fig. 8 (a)** shows that the range of static instability deceased by 95%, while the stability region expanded with the first critical velocity.

We also note that the biggest change in natural frequencies corresponds to the first natural frequency with an affinity ratio 41%. Otherwise, according to the **Fig. 8 (b)**, the frequencies decrease when increasing the length by about 150%, where the highest percentage decease is in the third frequency by 16.66%. **Fig. 8 (b)** and **Fig. 8 (c)** show the disappearance of the instability range, which corresponds to the disappearance of dynamic instability. In the same manner, the static critical velocity decreased, reducing stability region. The figure 9 provides roughly the same results for the **Fig. 8 (c)** as the mass ratio is 0.5.

The **Fig. 10** shows critical velocities that correspond to static instability of clamped-clamped pipe as function of the masse ratio in the field to [7] for different lengths and without elastic foundation. Results show that the increase in length reduces the critical velocity value. We notice that the critical speed range is very large between L = 2 m and L= 2.5 m, while the value are lower, when the length belongs to [5-10].

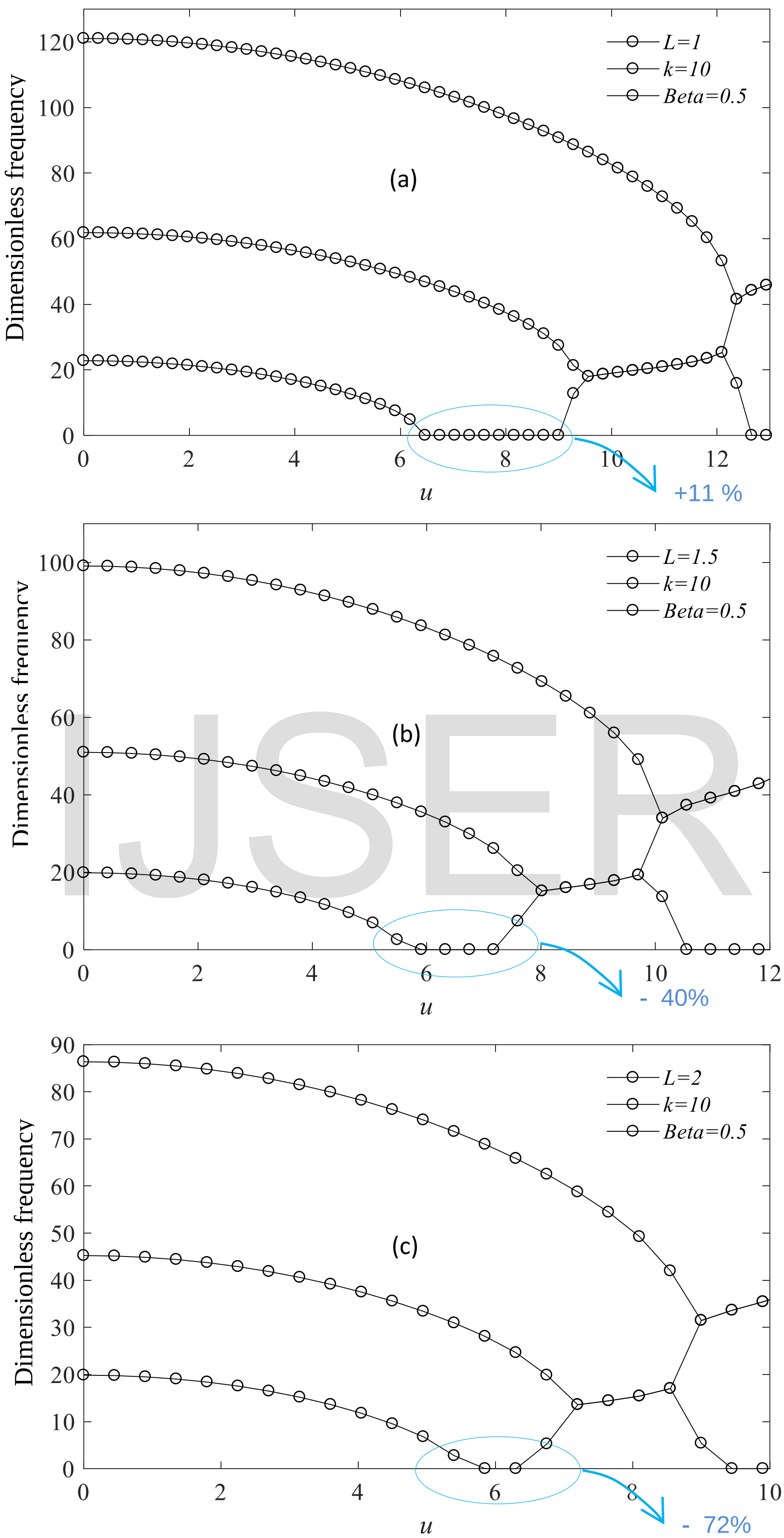


Fig. 7 Effect of length on the natural frequency of the clamped-clamped pipe on elastic foundation (k = 10) at different fluid velocities, β = 0.5.

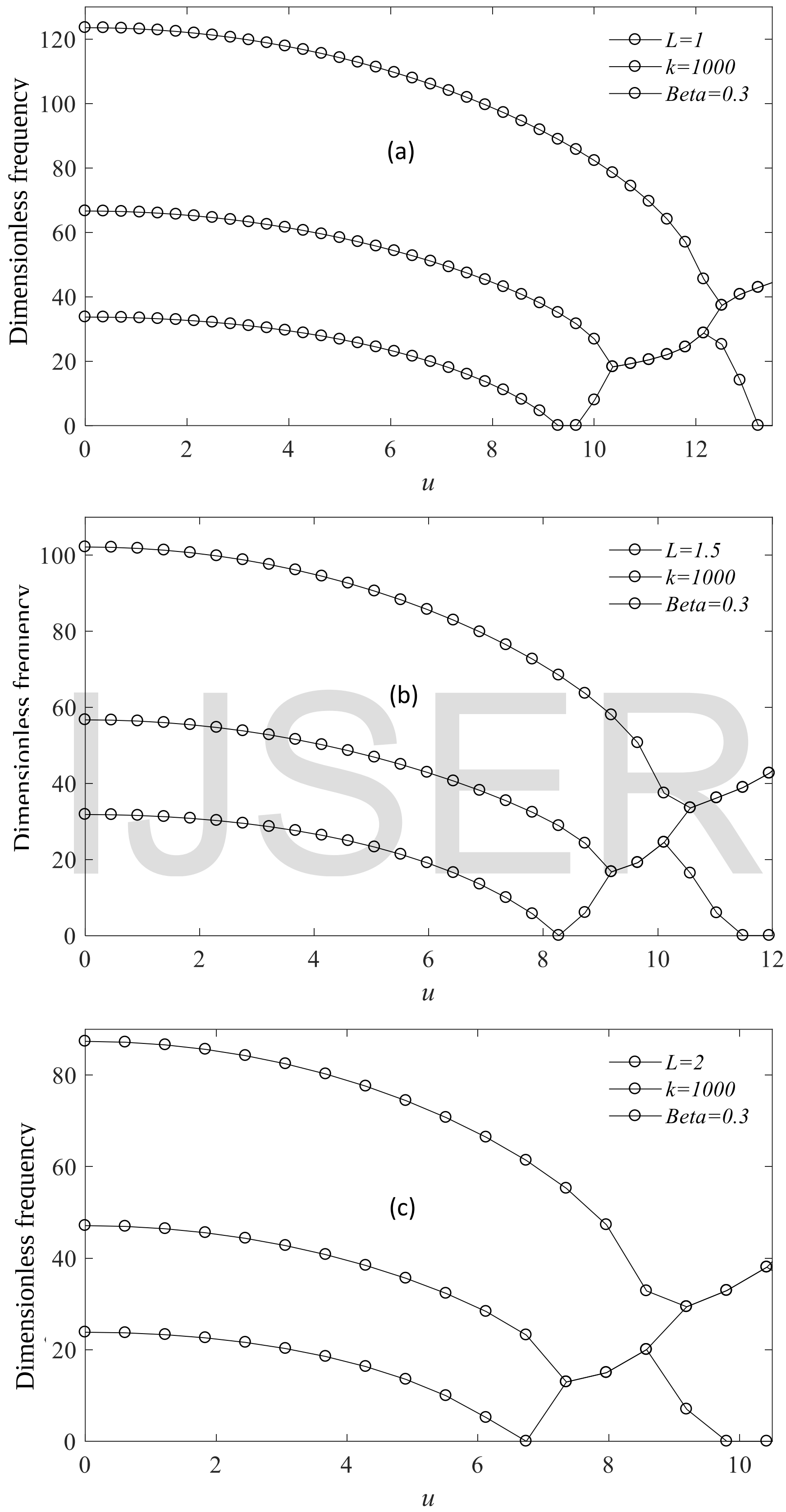


Fig. 8 Effect of length on the natural frequency of the clamped-clamped pipe on elastic foundation (k = 1000) at different fluid velocities, β = 0.3.

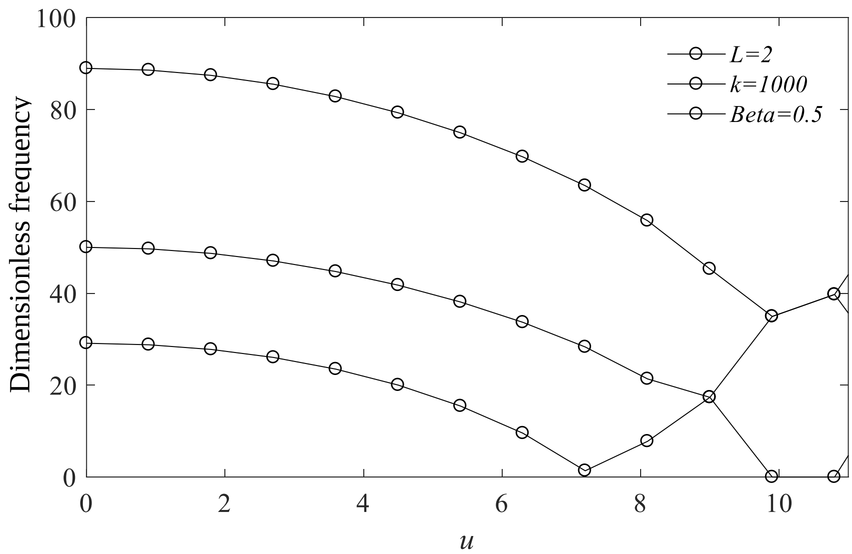


Fig. 9 Effect of length on the natural frequency of the clamped-clamped pipe on elastic foundation (k = 1000) at different fluid velocities, β = 0.5.

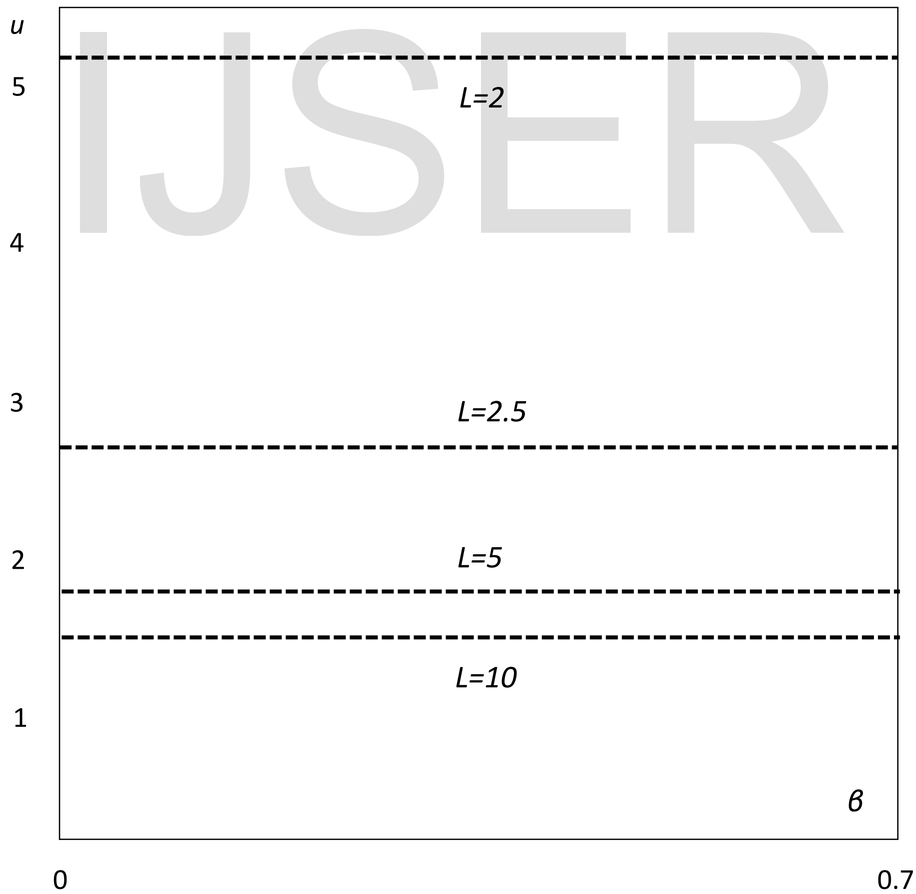


Fig. 10 Critical velocities of clamped-clamped pipes as a function of mass ratios for different lengths.

Conclusion

In this work, we have studied the static instability and dynamic instability of clamped-clamped pipe under internal fluid. Numerical modeling, which is seen as a promising tool for the design of pipes, on condition of being of an affordable calculation cost. In order to show the quality of the modilization by the finite element method by Matlab, the results of our numerical calculations are in qualitative agreement with the semi-analytical results (DTM). The numerical study allowed us to obtain a very good precision by using the element of beam; each node contains two degrees of freedom. Several examples have been treated for the study of the influence of different geometric and physic parameters on the system instability. We observe that instability appears when the velocity exceeds a threshold called critical velocity of instability, when the first natural pulse disappears. The critical fluid flow velocity varies as a function of the mass ratio, and reflecting the stability region of our system, while the Winkler type elastic foundation increases the rigidity of the system and therefore the critical instability velocities, while the range of static instability is decreasing. The increase of mass ration is accompanied by expansion in the stability region. We have noticed that increasing length L slightly decreases the natural frequencies of the system and consequently decreases their critical velocities, and leads to an increase in the instability range.

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Appendix A

The different elementary matrices for our system can be represented as follows,

$$[K_s] = \frac{m_f U^2}{30L}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{12}$$

$$[K_f] = \frac{m_f U^2}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & 3L^2 & -3L & 4L^2 \end{bmatrix} \tag{13}$$

$$[M] = \frac{(m_s+m_f)L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{14}$$

$$[C] = \frac{2m_f U}{30}\begin{bmatrix} -30 & 6L & 30 & -6L \\ -6L & 0 & 6L & -L^2 \\ -30 & -6L & 30 & 6L \\ 6L & L^2 & -6L & 0 \end{bmatrix} \tag{15}$$

$$[F] = \frac{KL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{16}$$

Where [K_s], [K_f], [M], [C] and [F] respectively, the structure stiffness, the fluid stiffness, the masses, the damping and the foundation matrices of the system.

Appendix B

The matrices [E] and [G] are calculated through variable-change as the following,

$$E = \begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \tag{17}$$

$$G = \begin{bmatrix} C & K \\ -K & 0 \end{bmatrix} \tag{18}$$

If, the solution of equation (8) is taken as,

$$\{q\} = \{E\}.exp(\lambda t) \tag{19}$$

And, λ is eigenvalues of the system and the $\{E\}$ corresponding eigenvectors of this value,

$$\lambda = \omega j \tag{20}$$

And the solution of equation is sought in the general form,

$$z = \begin{Bmatrix} \lambda\{E\} \\ \{E\} \end{Bmatrix} exp(\lambda t) = \{\widetilde{E}\}\, exp(\lambda t) \tag{21}$$

The system equation of government can be transformed from state space by,

$$\left\{\begin{bmatrix} 0 & \boldsymbol{I} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} - \lambda \begin{bmatrix} \boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}\right\} \begin{bmatrix} \lambda\{E\} \\ \{E\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{22}$$

I is a unity matrix

We ask ourselves,

$$H = \begin{bmatrix} 0 & \boldsymbol{I} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{23}$$

Eigenvalues are complex; they give in the form,

$$\lambda^m = Re^m + j\omega^m \tag{24}$$

Re: the real part of the eigenvalue, and is the damping of our system.

ω : the imaginary part of the eigenvalue, is therefore the proper pulsation of system.